Energy-Efficient Power Allocation for Delay-Constrained Systems

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***Abstract*—In this paper, we obtain an energy-efficient power allocation technique for a Rayleigh block-fading channel with delay-limited applications. In particular, we consider a probabilistic delay constraint as the user quality-of-service (QoS) requirement, and incorporate the concept of effective capacity to obtain the maximum arrival rate, at which, the delay constraint is satisfied. We obtain the energy efficiency (EE), which is formulated as the ratio between the effective capacity and the total expenditure power, of this system and derive the power allocation strategy that maximizes the EE. Numerical results are conducted to corroborate our theoretical results. In addition, for comparison reasons, we plot the maximum achievable EE under two well-known power allocations schemes, namely, water-filling (*wf*) and constant power allocation (*cons*) when considering delay constraints. The results show that in stringent delay limited systems, adaptive power allocation improves the maximum achievable EE significantly.**

***Index Terms*—Cross-layer design, energy efficiency, delay constraint, effective capacity.**

I. INTRODUCTION

The expansion of wireless communications systems has been enabled through increasingly utilizing natural resources, such as power. In particular, to provide delay constraint, which is considered as one of the major user quality-of-service (QoS) requirements [1], high transmission powers are required [2]. However, the battery technology improvement rate is shown to be slower than the desire for increasing energy consumption in the communications systems [3]. Furthermore, the energy consumption of the information and communication technologies (ICT) accounts for almost 3% of the worldwide electric energy consumption [3], [4]. Therefore, energy efficiency (EE), which is defined as the data transferred per unit energy consumed, i.e., b/J, by the system, has recently received a growing attention [3]–[7].

The tradeoff between spectral efficiency (SE) and EE was first studied in [5], wherein the minimum bit energy required for transmitting one bit is obtained. In particular, the EE metric is considered as the ratio between the service rate (i.e., Shannon rate in [5]) and the transmission power. It is proved that the maximum EE is achieved at low signal-to-noise ratios (SNRs) or low spectral efficiency (SE). However, [5] does not account for the circuit-power consumption. In [6], the expenditure power is considered as a combination of a rate-independent circuit-power and transmission power, and joint power and subchannel allocation techniques are proposed for an Orthogonal Frequency-Division Multiple Access (OFDMA) channel. A water-filing (*wf*) power allocation approach is proposed in [7] to optimize EE, when a minimum-rate requirement is considered in the system. In addition, power allocation strategies for maximizing EE is provided in [8], wherein a concave-convex fractional programming approach is used.

The tradeoff between EE and SE is more restrained in delay limited systems, since more energy is required to satisfy the delay requirements while maintaining the total throughput of the system. Delay constraints are considered in [9], [10], wherein a power-delay tradeoff is studied by minimizing a weighted sum of the expenditure power and the average delay. An energy-efficient rate-control policy proposed in [11] considers a buffer-constrained system. On the other hand, delay-outage probability constraints are considered in [12], [13], wherein, EE is defined as the ratio between the effective capacity and the transmission power. The effective capacity provides a measure for the maximum constant arrival rate that can be transferred through the fading channel, provided that delay-outage probability requirement is satisfied. In addition, the minimum bit energy, which is defined as the average bit energy normalized by the effective capacity, is found in low-power systems [12].

In this paper, we find a power allocation strategy for achieving the maximum EE in systems with delay-limited applications. The EE metric is defined as the achievable effective-capacity-to-total-expenditure-power ratio when the user delay-outage requirement is satisfied. The expenditure power is considered as the combination of the circuit-power and the transmission power. We prove that the EE-maximization objective function is a concave-convex optimization problem, therefore a unique global maximum exists. By using fractional programming, we the find the optimum power allocation strategy to maximize the EE. For comparison reasons, we further compare the results of the power allocation technique proposed in this paper with the maximum achievable EE with *wf* and *cons* transmission techniques. We provide numerical results to investigate the effects of adaptive power allocation, circuit-power and delay constraint on the maximum achievable EE in a Rayleigh block-fading channel with delay constraints.

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II. System Model

In this paper, we consider a point-to-point wireless communication system. In the transmitter, the upper-layer packets are stored in the transmit first-in-first-out (FIFO) buffer at a constant arrival rate and will be read out and transmitted over the channel at a variable rate to be discussed in the following paragraphs.

Discrete-time Rayleigh block-fading channels are assumed for the link between the transmitter and the receiver. Channel gain is assumed to be a stationary and ergodic random process, which remains constant during a fading-block $T_{\rm f}$, but varies independently in the following fading-block. The transmitter updates its transmission power based on the channel state information (CSI) available at the transmitter prior to transmission.

We further assume that the stochastic service process $\{r[t], t = 1, 2, \ldots, T\}$ is ergodic and stationary. Given that the Gartner-Ellis theorem assumptions [14, Pages 34-36] are satisfied, then the effective capacity of an independent and identically distributed (i.i.d.) block-fading channel is given by [15]

$$E_c(\theta) = -\frac{1}{\theta T_{\rm f} B} \ln\left(\mathrm{E}\left[\mathrm{e}^{-\theta T_{\rm f} B r[t]}\right]\right) \quad \text{(b/s/Hz)}, \qquad (1)$$

where $\mathrm{E}[\cdot]$ indicates the expectation operator, t is the time-index of the fading-block, B denotes the transmission bandwidth. The service rate of the fading channel, $r[t]$, is

$$r[t] = \log_2\left(1 + p(\gamma[t], \theta)\gamma[t]\right) \quad \text{(b/s/Hz)}.$$

Here, $\gamma[t] = \frac{|h[t]|^2}{\sigma_{\rm n}^2 L_{\rm p}}$ is the channel-to-noise power ratio, with $|h[t]|^2$ indicating the unit-variance Rayleigh fading channel power gain, $L_{\rm p}$ is the distance-based path-loss, and $\sigma_{\rm n}^2 = N_0 B$, with N_0 indicating the noise spectral density. $p(\gamma[t], \theta)$ is the transmission power as a function of $\gamma[t]$, and θ. For the ease of notation, we refer to $\sigma_{\rm n}^2 L_{\rm p}$ by K_ℓ.

Assuming that the steady-state queue length process, $q(\infty)$, exists and based on the large deviation principle (LDP) theorem, it is shown in [16], [2] that

$$\Pr\{q(\infty) \geq x\} \approx \mathrm{e}^{-x\theta}, \qquad \text{for large } x.$$

where $\mathfrak{f}(x) \approx \mathfrak{g}(x)$ means that $\lim_{x\to\infty} \dfrac{\mathfrak{f}(x)}{\mathfrak{g}(x)} = 1$ and θ is found from (1) for a given arrival rate. Hereafter, θ will be referred to by delay exponent. Finally, the delay-outage probability can be approximated by [15]

$$P_{\rm delay}^{\rm out} = \Pr\{\text{Delay} \geq D_{\max}\} \approx \varepsilon \mathrm{e}^{-\theta\mu D_{\max}},$$

where μ is the maximum constant arrival rate in b/s/Hz when $\mu = E_c(\theta)$, $D_{\max}$ is the maximum tolerable delay in units of symbol duration or $\dfrac{1}{B}$, and ε is the probability of a non-empty buffer and can be approximated by the ratio of the constant arrival rate to the average service rate [16], [2].

It can be observed that $\theta \to 0$ corresponds to a system that can tolerate no delay constraint, while $\theta \to \infty$ refers to a system that can tolerate strict delay constraint. For a given θ, we can obtain the user maximum supportable arrival-rate under a given delay constraint. An interested reader is referred to [15] for more details.

Note that effective capacity relates to the asymptotic case for the delay and is defined for large values of $D_{\max}$. However, it has been shown in [2], [15] that this model also provides a good estimate for small values of $D_{\max}$.

III. Energy Efficiency in Delay Limited Systems

The problem for maximizing the effective capacity of a point-to-point block-fading channel under average transmit power constraints is studied in [2], which shows that high transmission powers are required for satisfying delay constraints while maintaining the throughput of the system. In this paper, we aim to maximize the transmitted data per unit energy, or equivalently, to minimize the energy consumption per bit, while the delay-outage constraint is satisfied.

The EE metric for delay limited systems is defined as the ratio between the effective capacity and the expenditure power [13], which can be formulated as[1]

$$\mathrm{EE}(\theta) = \frac{E_c(\theta, p(\gamma, \theta))}{P_{\rm c} + \mathrm{E}_\gamma[p(\gamma, \theta)]} \quad \text{(b/J/Hz)},$$

where $E_c(\theta, p(\gamma, \theta))$ indicates the effective capacity of the channel as a function of θ and $p(\gamma, \theta)$, $\mathrm{E}_\gamma[\cdot]$ indicates the expectation over γ, and $P_{\rm c}$ is the constant circuit-power that corresponds to the power dissipation of the transmitter circuitry, which is independent of the transmission rate [17].

The EE maximization problem, therefore, is formulated as

$$\begin{aligned} \mathrm{EE}_{\rm opt}(\theta) = \max_{p(\gamma,\theta)\geq 0} \; & \frac{-\dfrac{1}{\theta T_{\rm f} B} \ln\left(\mathrm{E}_\gamma\left[(1 + p(\gamma, \theta)\gamma)^{-\alpha}\right]\right)}{P_{\rm c} + \mathrm{E}_\gamma[p(\gamma, \theta)]} \\ \text{subject to:} \; & \mathrm{E}_\gamma[p(\gamma, \theta)] \leq P_{\rm av} \end{aligned} \qquad (2)$$

where $\alpha = \dfrac{\theta T_{\rm f} B}{\ln(2)}$ and $P_{\rm av}$ is the average transmit power limit. As one can observe, the maximization of the EE (2) involves the maximization of the ratio of two functions of $p(\gamma, \theta)$. In general, the EE maximization problem is different from SE maximization problems, as the transmit power constraint does not necessarily need to be satisfied with equality in EE optimal systems.

In the following, we first provide a solution for an unconstrained EE optimization problem given as

$$\mathrm{EE}_{\rm opt}^{\rm un}(\theta) = \max_{p(\gamma,\theta)\geq 0} \frac{-\dfrac{1}{\theta T_{\rm f} B} \ln\left(\mathrm{E}_\gamma\left[(1 + p(\gamma, \theta)\gamma)^{-\alpha}\right]\right)}{P_{\rm c} + \mathrm{E}_\gamma[p(\gamma, \theta)]}. \qquad (3)$$

[1] Hereafter, we omit the time index t, wherever, it is clear from the context.

It is been shown in [17] that when the achievable rate of the channel, e.g., the effective capacity in this paper, is a concave function of the transmit power, the EE maximization problem can be solved using fractional programming. We start by proving that the effective capacity formula, used in (3), is a concave function in $p(\gamma,\theta)$. The proof is provided in Appendix A. Therefore, the objective function in (3) is the ratio of a concave and an affine function in $p(\gamma,\theta)$, indicating hence that the EE objective function (3) is a concave-convex optimization function, and as such, we can obtain a solution for (3) by using fractional programming [18].

By using the transformation $\zeta = \dfrac{1}{P_{\rm c}+{\rm E}_\gamma[p(\gamma,\theta)]}$, the optimization problem (3) can be converted to

$$\min_{p(\gamma,\theta)\geq 0} \quad \zeta \ln\left({\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]\right) \tag{4}$$

$$\text{subject to: } \zeta\left(P_{\rm c}+{\rm E}_\gamma[p(\gamma,\theta)]\right)\leq 1. \tag{5}$$

The objective function in (4) is a concave function. Furthermore, since the denominator of (3) is an affine function [18], the inequality in (5) can be changed to equality. The Karush-Kuhn-Tucker (KKT) conditions are, therefore, both sufficient and necessary for the optimal solution [18]. We now provide the lagrangian of (4) according to

$$\begin{aligned}\mathcal{L}\left(p\left(\gamma,\theta\right),\zeta,\lambda\right) =& \zeta\ln\left({\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]\right),\\ &+\lambda\left(\zeta\left(P_{\rm c}+{\rm E}_\gamma[p(\gamma,\theta)]\right)-1\right),\end{aligned}$$

where λ is the Lagrangian parameter, and then expand the KKT conditions according to

$$\zeta\left(P_{\rm c}+{\rm E}_\gamma[p(\gamma,\theta)]\right)=1 \tag{6}$$

$$\zeta\frac{-\alpha\gamma\left(1+p(\gamma,\theta)\gamma\right)^{-\alpha-1}f_\gamma(\gamma)}{{\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]}+\zeta\lambda f_\gamma(\gamma)=0 \tag{7}$$

$$\ln\left({\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]\right)+\lambda\left(P_{\rm c}+{\rm E}_\gamma[p(\gamma,\theta)]\right)=0. \tag{8}$$

From (7), we get

$$\alpha\gamma\left(1+p(\gamma,\theta)\gamma\right)^{-\alpha-1}=\lambda{\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right].$$

Hence, the power allocation can be found as

$$p(\gamma,\theta)=\left[\frac{\alpha^{\frac{1}{1+\alpha}}}{\nu^{\frac{1}{1+\alpha}}\gamma^{\frac{\alpha}{1+\alpha}}}-\frac{1}{\gamma}\right]^+, \tag{9}$$

with $\nu=\lambda{\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]$ and $[x]^+=\max(0,x)$. The optimal value of λ can be found by inserting the power allocation (9) in (8), yielding

$$\begin{aligned}&{\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]\ln\left({\rm E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]\right)\\ &+\nu\left(P_{\rm c}+{\rm E}_\gamma\left[\left[\frac{\alpha^{\frac{1}{1+\alpha}}}{\nu^{\frac{1}{1+\alpha}}\gamma^{\frac{\alpha}{1+\alpha}}}-\frac{1}{\gamma}\right]^+\right]\right)=0.\end{aligned} \tag{10}$$

Note that (10) only depends on ν and is independent from ζ.

We can derive a closed-form expression for obtaining ν

$$\begin{aligned}&\nu\left(P_c+\left(\frac{\alpha K_\ell^\alpha}{\nu}\right)^{\frac{1}{1+\alpha}}\Gamma\left(\frac{1}{1+\alpha},\frac{K_\ell\nu}{\alpha}\right)-{\rm Ei}\left(\frac{K_\ell\nu}{\alpha}\right)\right)\\ &+k\left(\frac{K_\ell\nu}{\alpha}\right)\log\left(k\left(\frac{K_\ell\nu}{\alpha}\right)\right)=0,\end{aligned} \tag{11}$$

where ${\rm Ei}(x)=-\int_{-x}^{\infty}\frac{e^{-s}}{s}ds$ indicates the exponential integral, $\Gamma(a,x)=\int_x^\infty s^{a-1}e^{-s}ds$ is the upper incomplete Gamma function [19], and

$$k(y)=y^{\frac{\alpha}{1+\alpha}}\Gamma\left(\frac{1}{1+\alpha},y\right)+1-e^{-y}.$$

Let $P_{\rm un}$ denote the average transmit power at which the maximum EE can be achieved, which can be found by taking the expectation over the power allocation given in (9) with the optimal value for ν found from (11). One can show that the optimum power allocation strategy to maximize the EE, as obtained in (9), is also an optimum power allocation strategy to maximize the effective capacity of a Rayleigh fading channel with an average transmit power limit $P_{\rm un}$. In the following section, we use this fact to find the maximum achievable EE of a fading channel with average transmit power constraints.

IV. Maximum Achievable EE Under Transmit Power Constraints

Assume that we obtain the power allocation strategy for maximizing the power-unconstrained EE. In order to obtain the maximum power-constrained EE (2), we divide the optimization problem into two regions, i.e., when $P_{\rm av}>P_{\rm un}$ and when $P_{\rm av}\leq P_{\rm un}$.

A. When $P_{\rm av}>P_{\rm un}$

This condition refers to the case when the total transmission power for achieving the maximum unconstrained EE is lower than the transmit power limit. As such, the power allocation strategy for maximizing the unconstrained EE, also satisfies the average transmit power constraint in (2), and hence, the maximum power-constrained EE is the same as maximum power-unconstrained EE.

B. When $P_{\rm av}\leq P_{\rm un}$

This scenario refers to the case when the required power for maximizing the unconstrained EE is more than the input transmit power limit. Therefore, the EE optimization problem becomes an SE optimization problem, i.e., maximizing the effective capacity with an average input power constraint, as given below:

$$\begin{aligned}\max_{p(\gamma,\theta)\geq 0}\quad & -\frac{1}{\theta T_{\rm f}B}\ln\left({\rm E}_\gamma\left[e^{-\theta T_{\rm f}Br[t]}\right]\right),\\ \text{subject to:}\quad & {\rm E}_\gamma[p(\gamma,\theta)]\leq P_{\rm av}.\end{aligned} \tag{12}$$

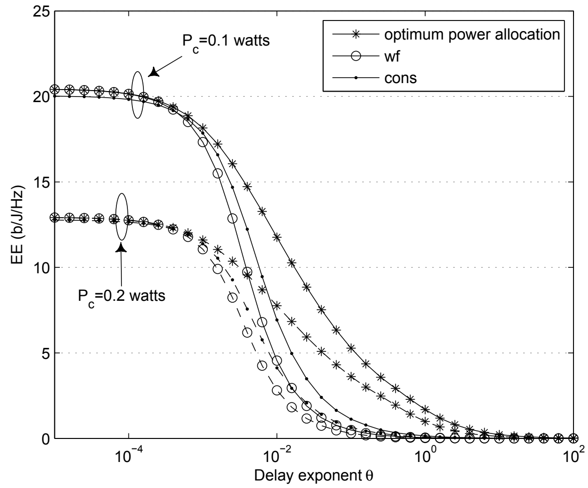

Fig. 1: Maximum achievable EE versus delay exponent, θ, for various circuit-powers, P_{c}s.

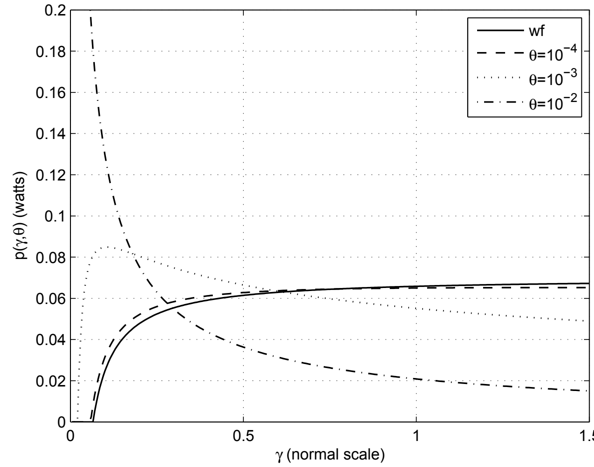

Fig. 2: Instantaneous transmission power $p(\gamma, \theta)$, given in (9), versus channel power gain for various delay exponents, θ.

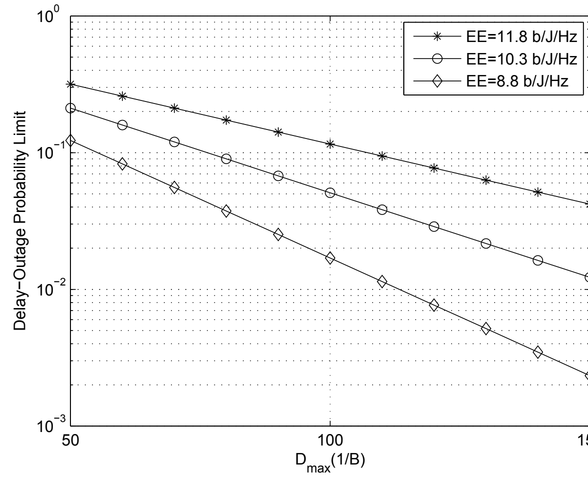

Fig. 3: Delay-outage probability versus maximum tolerable delay, $D_{\max}$, in units of symbol duration or $\dfrac{1}{B}$, for various EEs.

V. Numerical Results

In this section, we numerically evaluate the EE of a delay-limited system in Rayleigh block-fading channels. In the following numerical results, we assume the fading-block duration $T_{\mathrm{f}} = 2$ms, bandwidth $B = 180$ kHz, and noise spectral density $N_0 = -174$ dBm/Hz. Also, the distance-based path-loss for a macro-cell environment with a carrier frequency of 2 GHz is considered [20]

$$L_{\mathrm{p}}(\mathrm{dB}) = 128.1 + 37.6 \log_{10}(d),$$

where d is the distance between the transmitter and receiver and is considered to be $d = 1$km. The circuit-power is set to $P_{\mathrm{c}} = 0.1$ watts, otherwise only if indicated.

We start by plotting the graphs for the EE versus delay exponent, θ, for $P_{\mathrm{c}} = 0.1$ watts (solid lines) and $P_{\mathrm{c}} = 0.2$ watts (dashed lines), in Fig. 1. Fig. 1 also includes the plots for the maximum EE that can be achieved with the well-known *wf* and *cons* transmission techniques. In particular, in *cons* technique, the transmitter uses fixed power. The plots shows that the maximum achievable EE decreases as the delay exponent or the circuit-power increases. The optimum power allocation technique proposed in this paper achieves higher EE when compared to *wf* or *cons* techniques. In particular, while *wf* transmission technique performs good for loose delay limited systems, it achieves very low EE, even lower than *cons* technique, when delay becomes stringent.

In Fig. 2, we study the characteristics of the optimum power allocation technique, given in (9). Particularly, we plot the instantaneous transmission power, $p(\gamma, \theta)$, versus the channel power gain γ for various delay exponents. Fig. 2 shows that for delay limited systems with loose delay requirement, e.g., $\theta = 10^{-4}$, the power increases monotonically as γ increases, following the same trend as *wf*. However, as θ increases, the region wherein the transmission power is cut off shrinks and power is distributed to a wider range of channel power gains. By further increasing the delay exponent, more power is allocated to weaker channel gains. When delay becomes very stringent, maintaining a constant rate for a wide range of channel conditions, i.e., channel inversion transmission technique, is the optimal power allocation strategy.

In Fig. 3, we plot the delay-outage probability limit versus the maximum tolerable delay, $D_{\max}$, in units of symbol duration for various energy efficiencies. The figure shows that increasing the required EE results in increasing delay-outage probability significantly.

Finally, Fig. 4 includes the plots of the EE versus the input transmit power limit for various delay exponents. It shows that the EE benefits from increasing input transmit power limit for low signal-to-noise ratios. However, when the EE achieves its maximum point, increasing input transmit power limit does

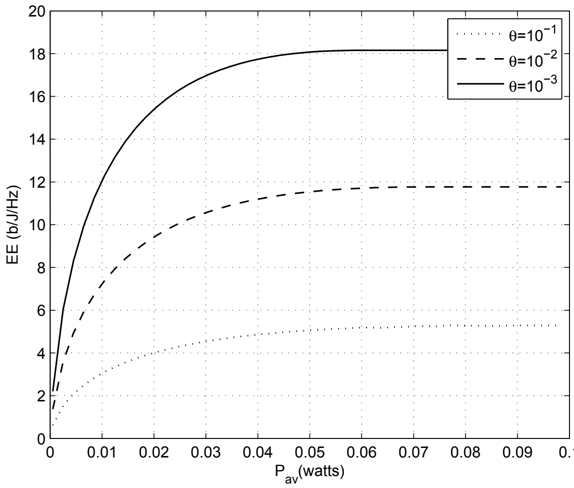

Fig. 4: Maximum achievable EE versus average transmit power limit for various delay exponents, θ.

not improve the achievable EE of the system.

VI. Conclusions

In this paper, we consider delay-outage constraints and find the maximum achievable EE in a Rayleigh block-fading channel. We incorporate the effective capacity concept that provides a measure for the maximum-arrival rate to maintain a delay-outage requirement. By proving that the EE maximization objective function is a concave-convex optimization problem in the transmission power, we provided a simple power allocation strategy for maximizing the EE of the channel through fractional programming. Illustrative results show that significant gains in the maximum achievable EE can be achieved through adaptively allocating the transmission power, specially in stringent delay limited systems.

Appendix A

We start by investigating whether the function

$$f(p(\gamma,\theta)) = \ln\left(U(p(\gamma,\theta))\right),$$

where $U(p(\gamma,\theta)) = \mathrm{E}_\gamma\left[(1+p(\gamma,\theta)\gamma)^{-\alpha}\right]$, is convex in $p(\gamma,\theta)$ or not. Hereafter, for the ease of notation, we refer to $p(\gamma,\theta)$ by P_t. We obtain the first partial derivative of $f(P_\mathrm{t})$ with respect to P_t , i.e., $f'(P_\mathrm{t})$, as follows:

$$\begin{aligned} f'(P_\mathrm{t}) &= \frac{U'(P_\mathrm{t})}{U(P_\mathrm{t})} \\ &= -\alpha\frac{\mathrm{E}_\gamma\left[\gamma(1+P_\mathrm{t}\gamma)^{-\alpha-1}\right]}{\mathrm{E}_\gamma\left[(1+P_\mathrm{t}\gamma)^{-\alpha}\right]} \\ &\leq 0, \end{aligned} \tag{13}$$

where $U'(P_\mathrm{t})$ is the first derivative of $U(P_\mathrm{t})$ with respect to P_t. Therefore, $f(P_\mathrm{t})$ is a non-increasing function of P_t. We further note that

$$f'(P_\mathrm{t})|_{P_\mathrm{t}\to\infty} \to 0. \tag{14}$$

We now derive the second partial derivative of $f(P_\mathrm{t})$ with respect to P_t, i.e., $f''(P_\mathrm{t})$, according to

$$\begin{aligned} f''(P_\mathrm{t}) &= \frac{U''(P_\mathrm{t})U(P_\mathrm{t}) - (U'(P_\mathrm{t}))^2}{U^2(P_\mathrm{t})} \\ &= \frac{g(P_\mathrm{t})}{U^2(P_\mathrm{t})}. \end{aligned}$$

where $U''(P_\mathrm{t})$ is the second derivative of $U(P_\mathrm{t})$ with respect to P_t and

$$\begin{aligned} g(P_\mathrm{t}) = {}& \alpha(\alpha+1)\mathrm{E}_\gamma\left[\gamma^2(1+P_\mathrm{t}\gamma)^{-\alpha-2}\right]\mathrm{E}_\gamma\left[(1+P_\mathrm{t}\gamma)^{-\alpha}\right] \\ & -\alpha^2\left(\mathrm{E}_\gamma\left[\gamma(1+P_\mathrm{t}\gamma)^{-\alpha-1}\right]\right)^2. \end{aligned}$$

We now prove that $g(P_\mathrm{t})$ is always positive. The value for $g(P_\mathrm{t})$ at $P_\mathrm{t}=0$ and $P_\mathrm{t}\to\infty$ can be obtained as

$$g(P_\mathrm{t})|_{P_\mathrm{t}=0} = \alpha(\alpha+1)\mathrm{E}_\gamma\left[\gamma^2\right] - \alpha^2\left(\mathrm{E}_\gamma\left[\gamma\right]\right)^2 \geq 0, \tag{15}$$

$$g(P_\mathrm{t})|_{P_\mathrm{t}\to\infty} \to 0,$$

where to obtain (15), we use the Jensen's inequality. We now prove that $g(P_\mathrm{r})$ can never cross the zero-line. We prove it by contradiction. Assume $g(P_\mathrm{r})=0$. It follows that

$$\begin{aligned} U''(P_\mathrm{t})U(P_\mathrm{t}) - (U'(P_\mathrm{t}))^2 = 0 &\Rightarrow \\ \frac{U''(P_\mathrm{t})}{U'(P_\mathrm{t})} = \frac{U'(P_\mathrm{t})}{U(P_\mathrm{t})} &\Rightarrow \\ \frac{\partial\ln(-U'(P_\mathrm{t}))}{\partial P_\mathrm{t}} = \frac{\partial\ln(U(P_\mathrm{t}))}{\partial P_\mathrm{t}} &\Rightarrow \\ \ln(-U'(P_\mathrm{t})) = \ln(U(P_\mathrm{t})) + c &\Rightarrow \\ U'(P_\mathrm{t}) = -e^c * U(P_\mathrm{t}) &\Rightarrow \\ \frac{U'(P_\mathrm{t})}{U(P_\mathrm{t})} = -e^c&. \end{aligned}$$

Therefore, $f''(P_\mathrm{t})$ can be zero only when $f'(P_\mathrm{t}) = -e^c$. We now analyze the pattern for $f'(P_\mathrm{t})$. Using (13) and (15), we note that $f'(P_\mathrm{t})$ starts from a negative value at $P_\mathrm{t}=0$ and increases until it reaches $f'(P_\mathrm{t}) = -e^c$. After this point, $f'(P_\mathrm{t})$ becomes a decreasing function. Noting that $f'(P_\mathrm{t})$ only changes direction when $f'(P_\mathrm{t}) = -e^c$, and using the fact that $f'(P_\mathrm{t})$ is a continuous function in P_t, one can show that $f'(P_\mathrm{t})$ will be always a decreasing function after it reaches $f'(P_\mathrm{t}) = -e^c$. This, however, contradicts with (14). Therefore, $g(P_\mathrm{t})$ can never cross the zero line. Given the fact that $g(P_\mathrm{t})|_{P_\mathrm{t}=0}$ is positive, we conclude that $g(P_\mathrm{t})$ is always positive and so is $f''(P_\mathrm{t})$. Therefore, the objective function $f(P_\mathrm{t})$ is convex and, as such, the effective capacity is a concave function of P_t.

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